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Nonrelativistic Green's Function for Systems With Position-Dependent Mass

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Given a spatially dependent mass, we obtain the 2-point Green's function for exactly solvable nonrelativistic problems. This is accomplished by mapping the wave equation for these systems into well-known exactly solvable Schrödinger equations with constant mass using point canonical transformation. The one-dimensional oscillator class is considered and examples are given for several mass distributions.

KEY WORDS: Green's function; position-dependent mass; point canonical transformation; effective mass; oscillator class.

1. INTRODUCTION

Quantum systems with spatially dependent effective mass were found to be very useful models for studying the physical properties of various microstructures and semiconductor interfaces in condensed matter. Special applications of these models are carried out in the investigation of electronic properties of semiconductors (Bastard, 1988), quantum wells and quantum dots (Harrison, 2000; Serra and Lipparini, 1997), ^{3}He clusters (Barranco *et al.*, 1997), quantum liquids (de Saavedra *et al.*, 1994), graded alloys and semiconductor heterostructures (see, for example, Einevoll, 1990; Einevoll *et al.*, 1990; Galbraith and Duggan, 1988; Gora and Williams, 1969; Morrow, 1987a,b; Trzeciakowski, 1988; Von Roos, 1983; Von Roos and Mavromatis, 1985; Weisbuch and Vinter, 1993; Young, 1989) . . . etc. These applications stimulated a lot of work in the literature on the development of methods and techniques for studying systems with mass that depends on position. Recently, several contributions have emerged that give solutions of the wave equation for such systems. The one-dimensional Schrödinger equation with smooth mass and potential steps was solved exactly by Dekar *et al.*, (1998, 1999). The formalism of supersymmetric quantum mechanics was extended to include

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position-dependent mass (Gönül *et al.*, 2002; Plastino *et al.*, 1999). Shape invariance was also addressed in this setting and the energy spectra were obtained for several examples. A class of solutions was obtained explicitly for such systems with equispaced spectra (Samani and Loran, 2003). Coordinate transformations in supersymmetric quantum mechanics were used to generate isospectral potentials for systems with position-dependent mass (Milanović and Iković, 1999). The ordering ambiguity of the mass and momentum operators and its effect on the exact solutions was addressed by de Souza Dutra and Almeida (2000) where several examples are considered. so(2,1) Lie algebra as a spectrum generating algebra and as a potential algebra was used to obtain exact solutions of the effective mass wave equation (B. Roy and P. Roy, 2002). Point canonical transformation (PCT) was recently used to obtain the energy spectra and wave functions for a large class of problems in one and three dimensions (Alhaidari, 2002). A class of quasi-exactly solvable problems with effective mass was presented by Koç *et al.* (2002) where the wave functions are obtained in terms of orthogonal polynomials satisfying mass dependent recurrence relation.

In all work cited above the main concern was in obtaining the energy spectra and/or wave functions for these systems once the position-dependent mass function is given. Moreover, exact solvability requirements result in constraints on the potential functions for the given mass distribution. On the other hand, the Green's functions for such systems, which are of prime significance in the calculation of physical processes, did not receive adequate attention. We are aware of only one contribution that dealt with Green's functions for systems with position-dependent mass: In 1995, Chetouani, Dekar, and Hammann used path integral formulation to relate the constant mass Green's function to that of position-dependent mass (Chetouani *et al.*, 1995). This was done on a formal level with explicit results in the two cases of step and rectangular-barrier potential and mass functions. In this article, we extend the PCT method used for obtaining the energy spectra and wave functions of such systems (Alhaidari, 2002) to the calculation of the 2-point Green's function. The basic idea behind the PCT method is as follows (see, for example, Bhattacharjie and Sudarshan, 1962; Goldstein, 1986; Junker, 1990; Manning, 1935; Montemayor, 1987; Pak and Sökmen, 1984). Starting with a problem whose solution (exact, quasi-exact, or conditionally exact) is known, then applying to it coordinate transformation that preserves the canonical form of the wave equation will map it into other solvable problems. The canonical constraint on the coordinate transformation generates classes of these solvable problems.

In Section 2, we start with the one-dimensional time-independent Schrödinger equation satisfied by the Green's function for a system with constant mass (the reference problem). Applying to it PCT maps it into the wave equation for the Green's function of a system with position-dependent mass. The canonical constraint defines the coordinate transformation in terms of the given mass function. It gives, as well, the potential functions for solvable systems with this position-dependent mass that belong to the class of the reference (constant mass) problem. A correspondence among the physical parameters of the two problems will also be generated. In Section 3, the formalism is implemented on the one-dimensional oscillator class and examples are given for several mass distributions.

2. ACTION OF THE PCT MAP ON THE GREEN'S FUNCTION

The momentum operator no longer commutes with the mass since the latter depends on position. In the majority of work done on the subject the following symmetric ordering of mass and momentum, in the kinetic energy part of the Hamiltonian, is adopted almost unanimously:

$$H = \frac{1}{2}\left[\vec{P}\frac{1}{M(\vec{r})}\vec{P}\right] + V(\vec{r}) = -\frac{\hbar^2}{2m_0}\left[\vec{\nabla}\frac{1}{m(\vec{r})}\vec{\nabla}\right]V(\vec{r}) \tag{2.1}$$

where $m(\vec{r})$ and $V(\vec{r})$ are real functions of the configuration space coordinates. Using atomic units ($m_0 = \hbar = 1$), this will result in the following time-independent wave equation in one dimension

$$\left\{\frac{d^2}{dx^2} - \frac{m'}{m}\frac{d}{dx} - 2m[V(x) - E]\right\}\phi(x) = 0 \tag{2.2}$$

where E is the energy eigenvalue and $m' \equiv dm/dx$. The Green's function (resolvent operator) g_E associated with this problem is formally defined as $(H - E)^{-1}$, where E does not belong to the discrete spectrum of the Hamiltonian H. It satisfies the following inhomogeneous equation:

$$\left\{\frac{d^2}{dx^2} - \frac{m'}{m}\frac{d}{dx} - 2m[V(x) - E]\right\}g_E(x, \bar{x}) = 2m\delta(x - \bar{x}) \tag{2.3}$$

On the other hand, the one-dimensional equation satisfied by the 2-point Green's function for a system with constant mass, potential function $\mathcal{V}$, and energy $\mathcal{E}$ reads

$$\left\{\frac{d^2}{dy^2} - 2[\mathcal{V}(y) - \mathcal{E}]\right\}\mathcal{G}_{\mathcal{E}}(y, \bar{y}) = 2\delta(y - \bar{y}) \tag{2.4}$$

We apply to this last equation the following transformation

$$y = q(x), \qquad \mathcal{G}_{\mathcal{E}}(y, \bar{y}) = p(x)g_E(x, \bar{x})p^*(\bar{x}) \tag{2.5}$$

If the result is a mapping into Eq. (2.3), then this transformation will be referred to as "PCT." Now the action of (2.5), for real functions, on Eq. (2.4) maps it into the following equation:

$$\left\{\frac{d^2}{dx^2} + \left(2\frac{p'}{p} - \frac{q''}{q'}\right)\frac{d}{dx} + \left(\frac{p''}{p} - \frac{q''}{q'}\frac{p'}{p}\right) - 2(q')^2[\mathcal{V}(q(x)) - \mathcal{E}]\right\} g_E(x,\bar{x})$$
$$= \frac{2(q')^2}{p(x)p(\bar{x})}\delta(q(x) - q(\bar{x}))$$

By identifying this with Eq. (2.3) and using the relation $q'\delta(q(x) - q(\bar{x})) = \delta(x - \bar{x})$, we obtain the following conditions on the transformation (2.5) to be a PCT:

$$p(x) = \sqrt{q'/m} \tag{2.6}$$

$$V(x) - E = \frac{(q')^2}{m}[\mathcal{V}(q) - \mathcal{E}] + \frac{1}{4m}[F(m) - F(q')] \tag{2.7}$$

where $F(z) = z''/z - \frac{3}{2}(z'/z)^2$. Given a position-dependent mass $m(x)$, eq. (2.7) is a constraint relating the potential function $V(x)$ to the transformation function $q(x)$ for a given class defined by the reference potential $\mathcal{V}(y)$. Therefore, for each choice of potential $V(x)$ there will be an associated PCT function $q(x)$ satisfying eq. (2.7). Once $q(x)$ is determined then so is $p(x)$ as it is given by eq. (2.6). Consequently, the Green's function $g_E(x,\bar{x})$ for the position-dependent mass system will be given by (2.5) in terms of the known reference Green's function $\mathcal{G}_{\mathcal{E}}(y,\bar{y})$ as

$$g_E(x,\bar{x}) = \sqrt{m(x)m(\bar{x})/q'(x)q'(\bar{x})}\,\mathcal{G}_{\mathcal{E}}(q(x), q(\bar{x})) \tag{2.8}$$

Moreover, a correspondence map will also be generated by eq. (2.7) relating the physical parameters of the reference problem (e.g., $\mathcal{E}$) to those of the variable mass problem (e.g., E).

Our strategy for solving the constraint eq. (2.7) is by choosing a PCT function $q(x)$ that will result in a position-independent term on the right side of Eq. (2.7), which will be identified with the constant energy term E on the left. To this end we consider the following two possibilities:

(a) The first is $(q')^2 = m$ giving the PCT function $q(x) = \tau\mu(x)$, where τ is a length scale positive parameter and

$$\mu(x) = (1/\tau)\int \sqrt{m(x)}\,dx \tag{2.9}$$

For a given mass distribution $m(x)$, this choice of PCT function, when substituted in Eq. (2.7), results in the following energy and potential function:

$$E = \mathcal{E}$$
$$V(x) = \mathcal{V}(\tau\mu(x)) + \frac{1}{8m(x)}G(m(x)) \tag{2.10}$$

where $G(z) = z''/z - \frac{7}{4}(z'/z)^2$. It will also give the following sought-after 2-point Green's function:

$$g_E(x, \bar{x}) = [m(x)m(\bar{x})]^{1/4}\, \mathcal{G}_E(\tau\mu(x), \tau\mu(\bar{x})) \tag{2.11}$$

(b) The second PCT function is obtained by taking $(q')^2\mathcal{V}(q) = \pm m/\sigma^2$, where σ is another real parameter. This choice gives $q(x) = R^{-1}(\tau\mu(x)/\sigma)$, where $R(y) = \int\sqrt{\pm\mathcal{V}(y)}\,dy$, and results in the following:

$$E = \mp 1/\sigma^2$$

$$V(x) = \mp\frac{\mathcal{E}/\sigma^2}{\mathcal{V}(q(x))} + \frac{1}{8m(x)}G(m(x)) \pm \frac{1}{8\sigma^2\mathcal{V}(q(x))}\left\{\frac{\mathcal{V}''(q(x))}{\mathcal{V}(q(x))} - \frac{5}{4}\left[\frac{\mathcal{V}'(q(x))}{\mathcal{V}(q(x))}\right]^2\right\}$$

$$g_E(x, \bar{x}) = \sigma[m(x)m(\bar{x})\mathcal{V}(q(x))\mathcal{V}(q(\bar{x}))]^{1/4}\mathcal{G}_{\mathcal{E}}(q(x), q(\bar{x})) \tag{2.12}$$

where $\mathcal{V}' \equiv d\mathcal{V}(q)/dq$. Requiring that the frst term in the potential expression above be independent of energy (through σ) will result in a constraint that relates the parameter σ to the reference energy $\mathcal{E}$. However, the last term in $V(x)$ will always be independent of σ. This is due to the fact that this term comes from $F(q')$ in the general relation (2.7), which is homogeneous in q with zero degree.

It is to be noted, however, that other choices of $q(x)$ might also be found that could produce a constant term on the right-hand side of Eq. (2.7), thus resulting in other solutions. However, we are contented here with the two classes of solutions obtained above. In the following section we use this development to obtain the nonrelativistic 2-point Green's function for several systems with different position-dependent mass that belong to the oscillator class.

3. OSCILLATOR CLASS GREEN'S FUNCTIONS

In this section we apply the PCT method development above to obtain the nonrelativistic 2-point Green's function for several systems with position-dependent mass in the oscillator class where $\mathcal{V}(y) = \frac{1}{2}\omega^4 y^2$ and ω is the oscillator frequency. In this case, the PCT choice $(q')^2 = m$ gives $q(x) = \omega^{-1}\mu(x)$, where $\mu(x)$ is the dimensionless integral in (2.9) with the length scale parameter τ taken equals to $1/\omega$. The potential function obtained using Eq. (2.10) is

$$V(x) = \frac{1}{2}\omega^2\mu(x)^2 + \frac{1}{8m(x)}G(m(x)) \tag{3.1}$$

On the other hand, the PCT choice $(q')^2\mathcal{V}(q) = m/\sigma^2$ gives $q(x)^2 = (2\sqrt{2}/\sigma\omega^3)\mu(x)$ and results in the following potential function by using Eq. (2.12)

$$V(x) = -\frac{\mathcal{E}/\sqrt{2}\sigma}{\omega\mu(x)} + \frac{1}{8m(x)}G(m(x)) - \frac{3}{32}\frac{\omega^2}{\mu(x)^2}$$

To eliminate the energy dependence in the first term of this potential, we require that σ be linearly proportional to $\mathcal{E}$. From dimensional arguments and using the available parameters in the problem, we redefine σ as $\sigma = (\sqrt{2}/\lambda\omega^3)\mathcal{E}$, where λ is a dimensionless real parameter. Consequently, for this PCT choice, which now reads $q(x)^2 = (2\lambda/\mathcal{E})\mu(x)$, Eq. (2.12) gives the following energy, potential, and Green's function:

$$\begin{aligned} E &= -\frac{1}{2}\lambda^2\omega^6/\mathcal{E}^2 \\ V(x) &= -\frac{\lambda}{2}\omega^2\mu(x)^{-1} - \frac{3}{32}\omega^2\mu(x)^{-2} + \frac{1}{8m(x)}G(m(x)) \\ g_E(x,\bar{x}) &= \sqrt{2\omega}(-2E)^{-1/4}[m(x)m(\bar{x})\mu(x)\mu(\bar{x})]^{1/4}\mathcal{G}_{\mathcal{E}}(q(x),q(\bar{x})) \end{aligned} \tag{3.2}$$

Now to proceed beyond this point, we need to compute the reference Green's function $\mathcal{G}_{\mathcal{E}}(y,\bar{y})$ for the constant mass one-dimensional oscillator. This Green's function is well known. For a recent review, one may consult the work of Šamaj *et al.* (2002). In one of its representations, we could write it as

$$\mathcal{G}_{\mathcal{E}}(y,\bar{y}) = \frac{1}{W_{\mathcal{E}}}\psi_{\mathcal{E}}^{-}(y_<)\psi_{\mathcal{E}}^{+}(y_>) \tag{3.3}$$

where $y_>$ $(y_<)$ is the larger (smaller) of y and $\bar{y}$, $\psi_{\mathcal{E}}^{\pm}(y)$ are two independent solutions of the Schrödinger wave equation $\{d^2/dy^2 - 2[\mathcal{V}(y) - \mathcal{E}]\}\psi_{\mathcal{E}}^{\pm}(y) = 0$ which are regular at the boundary limits of $y_>$ $(y_<)$, respectively. The Wronskian $W_{\mathcal{E}}$ of these two solutions is written as

$$W_{\mathcal{E}} = \psi_{\mathcal{E}}^{+}(y)\frac{d\psi_{\mathcal{E}}^{-}(y)}{dy} - \psi_{\mathcal{E}}^{-}(y)\frac{d\psi_{\mathcal{E}}^{+}(y)}{dy} \tag{3.4}$$

which is independent of y as can be verified by differentiating with respect to y and using the wave equation. The explicit form of $\mathcal{G}_{\mathcal{E}}(y,\bar{y})$ depends on whether the one-dimensional configuration space is taken to be the whole real line $y \in (-\infty,+\infty)$ or only half the line $y \in (0,+\infty)$ (Šamaj *et al.*, 2002). For the whole line it reads

$$\begin{aligned} \overset{\Leftrightarrow}{\mathcal{G}}_{\mathcal{E}}(y,\bar{y}) = {} & \frac{2}{\sqrt{\pi}\omega^2}\Gamma\left(\frac{3}{4} - \frac{\mathcal{E}}{2\omega^2}\right)\frac{1}{\sqrt{y\bar{y}}} \\ & \times\left[\mathcal{M}_{\mathcal{E}/2\omega^2,1/4}(\omega^2 y_<^2) + \frac{1/4}{\sqrt{\pi}}\Gamma\left(\frac{1}{4} - \frac{\mathcal{E}}{2\omega^2}\right)\mathcal{W}_{\mathcal{E}/2\omega^2,1/4}(\omega^2 y_<^2)\right] \\ & \times\mathcal{W}_{\mathcal{E}/2\omega^2,1/4}(\omega^2 y_>^2) \end{aligned} \tag{3.5}$$

However, in the case of the semi-infinite real line, $(y \geq 0)$ we write it as follows:

$$\overset{\Rightarrow}{\mathcal{G}}_{\mathcal{E}}(y, \bar{y}) = \frac{2}{\sqrt{\pi}\omega^2}\Gamma\left(\frac{3}{4} - \frac{\mathcal{E}}{2\omega^2}\right)\frac{1}{\sqrt{y\bar{y}}}\mathcal{M}_{\mathcal{E}/2\omega^2,1/4}(\omega^2 y_<^2)\mathcal{W}_{\mathcal{E}/2\omega^2,1/4}(\omega^2 y_>^2) \tag{3.6}$$

where Γ is the gamma function, $\mathcal{M}_{a,b}$ and $\mathcal{W}_{a,b}$ are the Whittaker functions of the first and second kind, respectively (Bateman and Erdélyi, 1953; Buchholz, 1969; Gradshtein and Ryzhik, 1980; Magnus *et al.*, 1966). They are defined in terms of the confluent hypergeometric functions as

$$\begin{aligned} \mathcal{M}_{a,b}(z) &= z^{b+1/2}\, e^{-z/2}\, {}_1F_1(b - a + \frac{1}{2}; 2b + 1; z) \\ \mathcal{W}_{a,b}(z) &= \frac{\Gamma(-2b)}{\Gamma(\frac{1}{2} - b - a)}\mathcal{M}_{a,b}(z) + \frac{\Gamma(2b)}{\Gamma(\frac{1}{2} + b - a)}\mathcal{M}_{a,-b}(z) \end{aligned} \tag{3.7}$$

Substituting the above reference Green's functions (3.5) and (3.6) into either one of the two formulas for $g_E(x, \bar{x})$ in Eq. (2.11) or (3.2) gives the sought-after Green's functions. That is, we end up with the following two possibilities:

(a) $q(x) = \omega^{-1}\mu(x)$:

$$\left.\begin{aligned} V(x) &= \tfrac{1}{2}\omega^2\mu(x)^2 + U(x) \\ \overset{\Leftrightarrow}{g}_E(x, \bar{x}) &= \tfrac{2}{\sqrt{\pi}}\Gamma\left(\tfrac{3}{4} - \tfrac{E}{2\omega^2}\right)\tfrac{[m(x)m(\bar{x})]^{1/4}}{\omega\sqrt{\mu(x)\mu(\bar{x})}} \\ &\quad\times\Big[\mathcal{M}_{E/2\omega^2,1/4}(\mu(x_<)^2) + \tfrac{1/4}{\sqrt{\pi}}\Gamma\left(\tfrac{1}{4} - \tfrac{E}{2\omega^2}\right) \\ &\quad\times\mathcal{W}_{E/2\omega^2,1/4}(\mu(x_<)^2)\Big]\mathcal{W}_{E/2\omega^2,1/4}(\mu(x_>)^2) \\ \overset{\Rightarrow}{g}_E(x, \bar{x}) &= \tfrac{2}{\sqrt{\pi}}\Gamma\left(\tfrac{3}{4} - \tfrac{E}{2\omega^2}\right)\tfrac{[m(x)m(\bar{x})]^{1/4}}{\omega\sqrt{\mu(x)\mu(\bar{x})}} \\ &\quad\times\mathcal{M}_{E/2\omega^2,1/4}(\mu(x_<)^2)\mathcal{W}_{E/2\omega^2,1/4}(\mu(x_>)^2) \end{aligned}\right\} \tag{3.8a}$$

where $U(x) = \frac{1}{8m(x)}[m''/m - \frac{7}{4}(m'/m)^2]$

(b) $q(x)^2 = 2(\lambda/\mathcal{E})\mu(x)$:

$$\left.\begin{aligned} V(x) &= -\tfrac{\lambda}{2}\omega^2\mu(x)^{-1} - \tfrac{3}{32}\omega^2\mu(x)^{-2} + U(x) \\ \overset{\Leftrightarrow}{g}_E(x, \bar{x}) &= \tfrac{4}{\sqrt{\pi}}\Gamma\left(\tfrac{3}{4} - \tfrac{\lambda}{\alpha_E}\right)\tfrac{[m(x)m(\bar{x})]^{1/4}}{\omega\alpha_E} \\ &\quad\times\Big[\mathcal{M}_{\lambda/\alpha_E,1/4}(\alpha_E\mu(x_<)) + \tfrac{1/4}{\sqrt{\pi}}\Gamma\left(\tfrac{1}{4} - \tfrac{\lambda}{\alpha_E}\right) \\ &\quad\times\mathcal{W}_{\lambda/\alpha_E,1/4}(\alpha_E\mu(x_<))\Big]\mathcal{W}_{\lambda/\alpha_E,1/4}(\alpha_E\mu(x_>)) \\ \overset{\Rightarrow}{g}_E(x, \bar{x}) &= \tfrac{4}{\sqrt{\pi}}\Gamma\left(\tfrac{3}{4} - \tfrac{\lambda}{\alpha_E}\right)\tfrac{[m(x)m(\bar{x})]^{1/4}}{\omega\alpha_E} \\ &\quad\times\mathcal{M}_{\lambda/\alpha_E,1/4}(\alpha_E\mu(x_<))\mathcal{W}_{\lambda/\alpha_E,1/4}(\alpha_E\mu(x_>)) \end{aligned}\right\} \tag{3.8b}$$

where $\alpha_E = 2\sqrt{-2E}/\omega$.

Table I. The Mass Function $m(x)$, the Corresponding Integral $\mu(x)$, and the Potential Component $U(x) = \frac{1}{8m(x)}[m''/m - \frac{7}{4}(m'/m)^2]$ for Each of the Four Examples Mentioned at the End of Section 3

$m(x)$	$\mu(x)$	$U(x)$
$\left[\frac{\gamma+(\omega x)^2}{1+(\omega x)^2}\right]^2$	$\omega x + (\gamma - 1)\tan^{-1}(\omega x)$	$\omega^2\left(\frac{\gamma-1}{2}\right)\frac{3(\omega x)^4+2(2-\gamma)(\omega x)^2-\gamma}{[\gamma+(\omega x)^2]^4}$
$1+\tan\text{h}(\omega x)$	$\sqrt{2}\tan\text{h}^{-1}[\sqrt{1+\tan\text{h}(\omega x)}/\sqrt{2}]$	$-\frac{\omega^2}{32}\frac{7+\tan\text{h}(\omega x)}{[\sin\text{h}(\omega x)+\cos\text{h}(\omega x)]^2}$
$[\gamma+(\omega x)^2]^{-1}$	$\ln[\omega x+\sqrt{\gamma+(\omega x)^2}]$	$-\frac{\omega^2}{8}\frac{(\omega x)^2+2\gamma}{(\omega x)^2+\gamma}$
$\tan\text{h}(\omega x)^2$	$\ln[\cos\text{h}(\omega x)]$	$-\frac{\omega^2}{2}[\sin\text{h}(\omega x)^{-2}+(5/4)\sin\text{h}(\omega x)^{-4}]$

Note. ω is the length scale parameter and γ is a dimensionless parameter.

Finally, we give several examples in a tabular form (Table I) for systems with different position-dependent mass. For each system we write down the dimensionless integral $\mu(x)$, which is needed for the calculation of the potentials and Green's functions in (3.8a) and (3.8b). The table also lists the potential component $U(x)$ for each system. Our criterion for the selection of these mass distributions is that the square root of $m(x)$ is analytically integrable so that $\mu(x)$ is easily attainable by using Eq. (2.9). Moreover, we made an attempt to include mass functions that are frequently used in the literature. The mass distribution in the first example was studied by Plastino *et al.* (1999), Gönül *et al.* (2002), and B. Roy and P. Roy (2002). The second example represents a smooth mass step that becomes abrupt

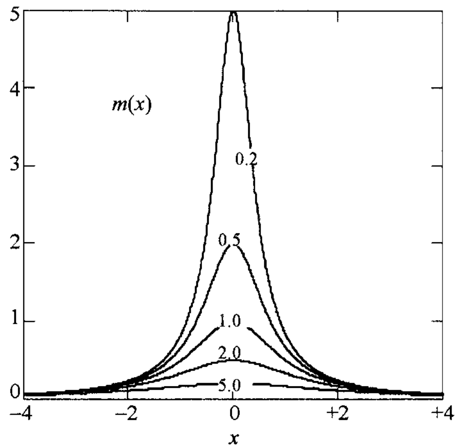

Fig. 1. The mass as a functions of position for the third example (third row in Table I). In the figure $\omega = 1.0$ (in arbitrary units) and the dimensionless parameter is assigned the values $\gamma = 0.2, 0.5, 1.0, 2.0, 5.0$, which are shown on their respective traces.

as ω becomes large. This example was treated by Dekar *et al.* (1998, 1999) but for a potential that has the same shape of a smooth step. Here, exact solvability gives two systems with this mass step but for potentials that differ from the one in the work of Dekar *et al.* (1998, 1999). Example 3 is for asymptotically vanishing mass with a maximum value of $1/\gamma$ at the origin. The mass in the fourth example is asymptotically flat with the value $m = 1$ but with a dip in the neighborhood of the origin. We show graphically, in Figs. 1 and 2, the mass and the two potential

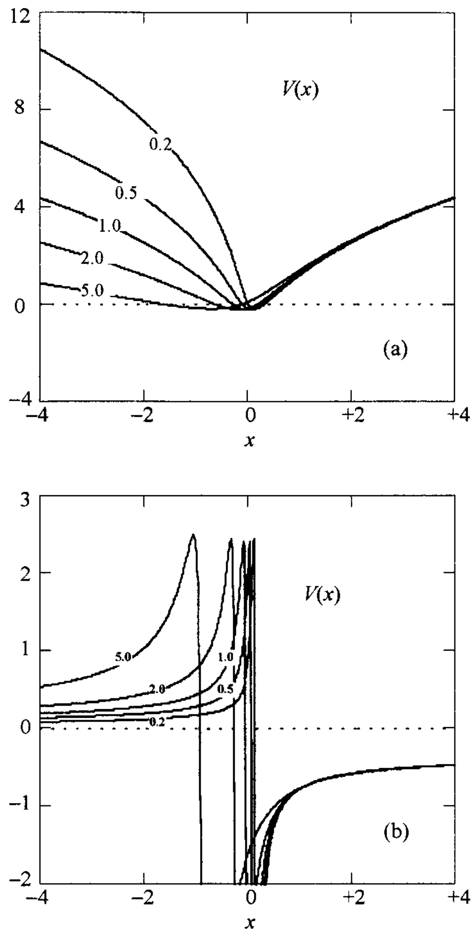

Fig. 2. The two potential function for the third example with the parameter values given in the caption of Fig. 1. The potential functions given by Eq. (3.8a)/Eq. (3.8b) was used to produce Figs. 2(a) and (b), respectively. We took $\lambda = 2.0$ in Fig. 2(b).

functions of the third example for several values of the dimensionless parameter γ while keeping the other parameters fixed at $\omega = 1.0$ and $\lambda = 2.0$.

In conclusion we should point out that finding the Green's function for other potential classes is also possible. The Coulomb, Morse, Scarf, and Pöschl–Teller, potentials are among such classes. The three-dimensional problem could as well, be treated using the PCT method as outlined in our earlier work (Alhaidari, 2002). Furthermore, the same formalism could, in principle, be extended to include non-analytic potential classes.

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